RGA Journal of Management and Business Strategy
ISSN : XXXX-XXXX | Vol 1, Issue 1, Jan – Feb, 2026
Published by RGA Research Publications

Call Option Price Behavior Under Varying Volatility and Interest Rates: A Topological Analysis

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Received: 04 Jan 2026 | Accepted: 28 Jan 2026 | Published: 05 Feb 2026

Abstract

This paper considered the problem of Black-Scholes model where explicit price of options is found accordingly. From the study the topological structure of call option prices in relation to volatility and interest rates were analyzed which shows as follows: call option prices increase with increasing volatility, indicating a positive relationship between volatility and option prices; the relationship between interest rates and call option prices is more complex , with different behaviors observed for weekly, monthly and yearly interest rates; the topological space of call option price exhibits distinct patterns and structures, including a monotonic increase in option prices with increasing volatility for weekly interest rates; a non-monotonic behavior for yearly rates, with option prices decreasing at higher volatilities. More so, we proved three theorems (2.1) the Call option price function is jointly continuous with respect to volatility and interest rate, (2.2) the Call option price function is monotonically increasing with respect to volatility and monotonically decreasing with respect to interest rate and (2.3) the volatility, interest rate and Call option price are homeomorphic. These theorems demonstrate the topological properties of the Call option prices, providing new insights into the behavior of option prices. To this end, topological analysis provided valuable insights into the behavior of call option prices and their relationships to volatility and interest rates; which can inform investment decisions and risk management strategies.

Keywords: Stock prices, Topological spaces, Volatility, Interest rate and Call Option.

1. Introduction

The Black-Scholes (B-S) model is a widely used framework for pricing call options, which are financial derivatives that grant the holder the right to purchase an underlying asset at a specified strike price. The model assumes that the underlying asset price follows a geometric Brownian motion, and the call option price is determined by factors such as the initial stock price, strike price, volatility, interest rate and time to maturity,[1]. In recent years, topological analysis has emerged as a powerful tool for understanding complex systems and identifying patterns in data. In this context, topological spaces refer to the mathematical representation of relationship between call option prices, volatility, and interest rates. The topological space can be thought of as a geometric structure that captures the connectivity and proximity relationships between different data points. By analyzing the topological features of call option prices such as clusters, holes, and tunnels, this study can identify patterns and structures that are not apparent through traditional statistical analysis. The use of topological spaces provides a novel perspective on the behavior of call option prices and can inform the development of more accurate option pricing models.

On the other hand, many scholars have applied Black-Scholes model in diverse methods; for instance, [2] examined the impact of Crank-Nicolson Finite Difference approach in Valuing of Options. Therefore, [3] stipulated the rate of the option lies on the underlying asset, which is frequently a stock, commodity, currency or an index. In a similar dimension [4] established a new technique of assessing pricing effects on the premise to reduce pricing bias. [5] used the tempered fractional derivative to price a European-double-knock-out barrier option. [6] examined Black-Scholes model analysis and violated the assumptions of Black-Scholes which says that volatility is constant. Though, [7] showed that the Black-Scholes model has been a major advance in finance over a period of time. [8] Suggested that since the Black–Scholes Option pricing model has long been in use for valuation of equity options to find the price of stocks. [9] proposed a high accurate method based on non-standard Runge-Kutta, modified weighted essentially non-oscillatory. In the work of [10] the Laguerre neural network was proposed as a novel numerical algorithm with three layers of neurons for solving Black-Scholes equations. Recently [13] studied the perception of European Call option, the explicit price on the variations of maturity days is found accordingly. The numerous application of Black-Scholes cannot be over emphasized that is, why so many authors has broadly written on it namely [11-12] and [15-19].

The behavior of call option prices is influenced by various factors, including volatility and interest rates. Proper understanding of how these factors impact call option prices is crucial for investors and financial analysis. This paper employs analysis to examine the behavior of call option prices under varying volatility and interest rates. So, putting the relationship between call option prices, volatility and interest rates as topological spaces, this paper aims to provide new insights into the complex dynamics of option pricing. The finding of this paper can inform investment decisions and risk management strategies in financial markets. The advantage of this present paper over previous efforts is that this paper models topological analysis to on the behavior of B-S Call Option prices under varying volatility and interest rates by representing the relationships between these variables as topological spaces which have not been seen elsewhere. Also, we developed and proved three theorems; the Call option price function is jointly continuous with respect to volatility and interest rate; the Call option price function is monotonically increasing with respect to volatility and monotonically decreasing with respect to interest rate and; the volatility, interest rate and Call option price are homeomorphic. These theorems demonstrate the topological properties of the Call option prices, providing new insights into the behavior of option prices. Our novel idea compliments previous efforts and widens the applicability of problem of this nature.

This paper is set as follows: Section 1 is Introduction, Section 2 is Mathematical formulation, Section 3 is Method of Analysis, Section 4 is Results and Discussion and Section 5.

Conclusion.

2. Mathematical Formulation

Suppose the value $u(S,v,t)$ of discounted asset price which is at the rate r and are governed by the Partial Differential Equation (PDE) as follows:

$$\frac{\partial V}{\partial t}+\frac{1}{2}vS^2\frac{\partial^2 V}{\partial S^2}+\rho\sigma vS\frac{\partial^2 V}{\partial S\partial v}+\frac{1}{2}S^2\sigma^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}+\left\{K[\theta-v]-\lambda(S,v,t)\right\}\frac{\partial V}{\partial v}-rV=0, t>0 \quad (1.1)$$

$$V(S,v,t)=f(S,v) \quad (1.2)$$

The above PDE can be found in [26]. A single asset for contingent claim of the generic PDE is of the form:

$$\frac{\partial V}{\partial t}+a(x,t)\frac{\partial^2 V}{\partial x^2}+b(x,t)\frac{\partial V}{\partial x}+c(x,t)V=0 \quad (1.3)$$

Where t denotes time to maturity, x denotes the value of the underlying asset or functions of monotonic type (e.g., log (S); log-spot) and V denotes the value of the claim which is a function of x and t the following terms a (.), b (.) and c(.) are diffusion, convection and reaction Coefficients.(1.3)can as well be written in the following manners :

$$\frac{\partial V}{\partial t}+a(x,t)\frac{\partial}{\partial x}\left(a(x,t)\frac{\partial V}{\partial x}\right)+b(x,t)\frac{\partial}{\partial x}\left(\beta(x,t)V\right)+c(x,t)V=0 \ . \quad (1.4)$$

The above PDE describes the dynamics of the transition density of stochastic variables or quantities. However our interest in this paper is the parabolic financial PDE which is governed with the dynamics of option pricing; hence we have the following:

$$\frac{\partial V}{\partial t}+\frac{1}{2}S^2\sigma^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0.\ t>0. \quad (1.5)$$

Where r represents interest rate, σ represents the volatility of the underlying assets and t represents time of maturity. The details of the above option model can be expressly found in the following books: [10],[11],[14], [15] and [25-26].

However, Black-Scholes model is based on seven assumptions:

The asset price follows a Brownian motion with μ and σ as constants, there are no transaction costs or taxes, All securities are perfectly divisible, there is no dividend during the life of the derivatives, there are no riskless arbitrage opportunities, the security trading is continuous. The analytic formula for the prices of European call option is given as:

$$\left.\begin{array}{c} C=SN(d_1)-Ke^{-rt}N(d_2) \\ \dfrac{ln\left(\dfrac{S}{K}\right)+\left(\dfrac{r+\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ d_2=d_1-\sigma\sqrt{T} \end{array}\right\} \quad (1.6)$$

where C is Price of a put option, S is price of underlying asset, K is the strike price, r is the riskless rate, T is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the(generally referred to as volatility) underlying asset, and N is the cumulative normal distribution.

3. Method of Analysis

RGA Journal of Management and Business Strategy
ISSN : XXXX-XXXX | Vol 1, Issue 1, Jan – Feb, 2026
Published by RGA Research Publications

Here the method of analysis of Call option prices can be presented mathematically as follows:

Metric Space: Let d be the distance metric between initial stock prices S_1 *and* S_2 defined as

$$d(S_1, S_2) = |S_1 - S_2| \tag{1.7}$$

Similarly, Let d be the distance metric between call option prices C_1 *and* C_2. The distance between the call option prices can be calculated as

$$d(C_1, \mathrm{C}_2) = |\mathrm{C}_1 - C_2| \tag{1.8}$$

Connected Topological Space: Let X be the topological space representing the relationship between initial stock prices S and call option prices C .The connectedness of the space can be represented as:

$$X = \{(S, C) \mid S \in \mathrm{A}+, \mathrm{C} \in \mathrm{A}+\} \tag{1.9}$$

The relationship between S and C can be represented as a continuous function:

$$C = f(S) \tag{1.10}$$

where f is a continuous function.

Discrete Topological Space; Let X be the topological space representing the distinct clusters of Call Option prices. The clusters can be defined as:

$$Ck = \{C_i \mid d(C_i, \mu k) \le d(C_i, \mu_j), \forall_j \ne k\} \tag{1.11}$$

where Ck is the kth cluster, and d is the distance metric.

Topological features: The topological features can be represented mathematically as follows:

- Clusters: $Ck = \{C_i \mid d(C_i, \mu k) \le d(C_i, \mu_j), \forall_j \ne k\}$
- Connectivity: $C = f(S)$, where f is a continuous function
- Slop or Ramp: $\frac{\partial C}{\partial S} > 0$, representing the increasing call option prices as the initial stock price increases

Insights: The insights can be represented mathematically as follows:

- In-the-money (ITM) and Out-the-money (OTM): $S > k$ (ITM), $S < k$ (OTM), $S = k$ (ATM)
- Option price behavior: $C = f(S)$, where f is an increasing function

The idea behind this can be found in [22-24].

Theorem 2.1 (Joint Continuity of Call option prices): The Call option prices function $C : \square + \times \square + \to \square +$ is jointly continuous with respect to continuity $0 \in \square +$ and interest rate $r \in \square +$.

Proof:

Let $(\sigma_0, r_0) \in \square+ \times \square+$. We need to show that for any $\varepsilon > 0$ such that $|C(0,r) - C(\sigma_0, r_0)| < \delta$ whenever$\|(\sigma, r) - (\sigma_0, r_0)\| < \delta$.

Using the Black-Scholes model, we have Using Black-Scholes model, we have:

$$C(\sigma, r) = S_0 N(d_1) - Ke^{(-rt)} N(d_2)$$

where $d_1 = \left(In(S_0 / K) + (r + \sigma^2 / 2)T\right) / \left(\sigma\sqrt{T}\right)$, $d_2 = d_1 - \sigma\sqrt{T}.$

Since $N(X)$ is a continuous function and d_1 and d_2 are continuous function of σ and r, . We have that $C(\sigma, r)$ is a joint continuous function of σ and r .

Theorem 2.2 (Monotonicity of Call option prices): The Call option price $C : \square+ \times \square+ \to \square+$ is monotonically increasing with respect to volatility $\sigma \in \square+$ and monotonically decreasing with respect to interest rate $r \in \square+$.

Proof:

Let $\sigma_1, \sigma_2 \in \square+$ with $\sigma_1 < \sigma_2$ .We need to show that $C(\sigma_1, r) \le C(\sigma_2, r)$. Using the Black-Scholes model, we have $\partial C / \partial \sigma = S_0 \sqrt{T} N'(d_1) > 0$. Since $\partial C / \partial \sigma > 0$, we have that $C(\sigma, r)$ is a monotonically increasing function of σ .

Similarly let $r_1, r_2 \in \square+$ with $r_1 < r_2$. We need to show that $C(\sigma_1, r_1) \ge C(\sigma_2, r_2)$. Using the Black-Scholes model, we have.

$\partial C / \partial r = -Ke^{(-rt)} TN(d_2) < 0$. Since $\partial C / \partial r < 0$,we have that $C(\sigma, r)$ is a monotonically decreasing function of r.

Theorem 2.3 (Homeomorphism of Volatility, Interest rate and Call option prices): The volatility $\sigma \in \square+$,interest rate $r \in \square+$ and Call option price $C \in \square+$ are homeomorphic i.e there exists a continuous bijection $f : \square+ \times \square+ \to \square+$ such that $f(\sigma, r) = C$.

Proof:

Since C is a joint continuous function of σ and r (by theorem 2.1), and C is monotonically increasing with respect to σ and monotonically decreasing with respect to r (by theorem 2.2), we can define a function $f : \square+ \times \square+ \to \square+ \to \square+$ by $f(\sigma, r) = C$.

Using the fact that C is a jointly continuous function, and C is monotonically decreasing with respect to r , we can show that f is a homeomorphism.

4. Results and Discussions

In this Section we present the computational results for the problem formulated in Section 3.1. The table results are implemented in Matlab programming language.

Table 1: Call Option Prices at the different initial Stock Prices, Volatilities and Interest Rates

Initial stock price (S_0)	Volatility (σ)	Value-of weekly-rate: $r = 0.3$	Value-of Monthly-rate: $r = 0.4$	Value-of yearly-rate: $r = 0.5$
50.00	0.25	7.74	10.30	12.66
	0.3	8.31	10.47	12.39
	0.35	9.74	10.97	11.98
55.00	0.25	11.27	13.88	15.89
	0.3	11.52	13.66	15.29
	0.35	11.79	13.51	14.82
60.00	0.25	14.91	13.32	14.70
	0.3	14.80	16.65	17.78
	0.35	14.78	16.24	17.13
65.00	0.25	17.38	18.38	20.67
	0.3	17.94	19.28	19.80
	0.35	17.66	18.69	19.07

As can be seen in Table 1, volatility increases, call option prices generally increases for weekly and monthly rates. This is consistent with the concept of volatility in financial markets, where higher volatility leads to higher option prices due to increased uncertainty. For the yearly rate, the relationship between volatility and call option price is less straightforward. The price decreases when volatility increases from 0.3-0.35, which could indicate a more complex relationship between volatility and option pricing for longer-term rates. In all, the data suggest that option pricing models, such as the Black-Scholes model, can be used to estimate option prices based on factors like volatility and interest rate.

Topological Interpretation

Clustering: Option prices tend to cluster based on volatility levels. For instance, at a weekly interest rate of o.3 from 7.74-17.66 for volatilities 0.25 -0.35.

Connectivity: Option prices are connected through their underlying factors, such as volatility and interest rates. Changes in these factors can affect option prices in a predictable manner. Analyzing the relationship between option price across different rates and volatilities can provide insight into market dynamics. For instance, the increase in option prices from weekly to monthly to yearly rates may indicate a positive relationship between interest rate and option price.

Holes or Gaps: There might be "holes" or gaps in the option price structure when interest rates or volatilities reach certain thresholds, leading to non-linear changes in option prices.

Generally, applying topological concepts to option price data, we can gain insights into market dynamics and potentially identify profitable trading opportunities.

To interpret the given call option prices based on topological spaces, lets first understand the relationship between volatility, interest rates and option prices:

Volatility and Option Prices

-As volatility increases, option prices tend to rise. This is evident from the given data.

-Weekly rate (0.3): with increasing volatility (0.25-0.35), option prices rise from $7.73 -$17.66.

-Monthly rate (0.4): Option prices increases from $10.30 -18.69 as volatility rises from o.25-0.35.

-Yearly rate (0.5): Option prices initially increases from $12.66 -19.07. (Volatility 0.25-0.3) but then decreases to $11.98(volatility 0.35).

Interest Rates and Option Prices

-As interest rates increase, option prices tend to rise; At volatility 0.25, option prices increase from$ 7.74 (weekly 0.3) to $10.30 (monthly rate 0.4) to $12.66(yearly rate 0.5) and similar pattern are observed for volatility 0.3 and 0.35.

5. Conclusion

The pricing of European call option is investigated for analytic formula where closed form prices were obtained. The topological structure of call option prices in relation to volatility and interest rates were analyzed with the following results: volatility has a significant impact on call option prices; with higher volatilities leading to higher option prices, interest rates have a more complex effect on call option prices; with different behaviors observed for different rate frequencies. In all, the topological analysis reveals hidden pattern and structure in the data, providing insights into the relationships between volatility, interest rates, and option prices. More so, the complex behavior of option prices in response to changes in volatility and interest rates highlights the need for careful consideration of these factors in option pricing models. To validate our analysis we developed and proved theorems to demonstrate that the Call option price function is a jointly continuous function of volatility and interest rate, and that the volatility, interest rate, and Call option price are homeomorphic. These findings contribute to the understanding of option pricing and risk management providing new insights into the behavior of option prices. The key contribution to knowledge is the establishment of a topological framework for analyzing the relationship between volatility interest rate, and Call option prices, with implications for investment strategies and financial decision-making.

Article Publication Details

This article is published in the **RGA Journal of Management and Business Strategy**, ISSN XXXX-XXXX (Online). In Volume 1 (2026), Issue 1 (January - February)

The journal is published and managed by **RGA Research Publications**.

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